IOP Conf. Series: Journal of Physics: Conf. Series **1028** (2018) 012148 doi:10.1088/1742-6596/1028/1/012148

# Tortuous thinking intuitively in solving problem of sequence convergence

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**Abstract**. This paper describes the process of intuitive thinking in a tortuous way to solve problem of sequence convergence by students Program Study of Mathematics Education, Faculty of Teacher Training and Education of Universitas Lampung. It is said to be tortuous because the student experienced several stuck in solving the problem. This study was conducted on 30 students, and there are 2 students netted as research subjects. Data is obtained from audio visual recording of student's think aloud and interview. The tortuous thinking intuitively can be explained in the following order. Student 1) understands the problem self-evidently and accepts the existing statement in the issue with intrinsic certainty and with overconfidence, 2) plans the problem solving immediately and tends to rush, through trial and error based on primacy effect, 3) carries out the planning in order to get behavior of the sequence's term; however he is stuck. Then the students go back to step 2) plans the problem solving and then continues to step 3). This can last several times. After finding the behavior of the term of sequence, students goes to the next step that is 4) makes extrapolation about solution, and then 5) declares the solution with complete sentence.

## 1. Introduction

Convergence of sequence is one of items in Real Analysis lectures that must be understood in a complex and rigor. It is said complex because its definition contains many aspects such as absolute value inequality, existential and universal quantors, and functional relationship between ε and *N*. It is said to be rigor because students are required to analyze or synthesize, evaluate and validate a statement in mathematics creatively and develop a theorem associated with the Real number (eg the system of Real Number, sequence and series, as well as the metric space) and its application in the field of mathematics and other fields (Academic Guides of the University of Lampung). The rigorousness in learning Real Analysis, especially the convergence of sequence poses a problem. In order to reduce the problems caused by it, students should be given the opportunity to use their intuitive thinking as a decisive part in the acceptance of new knowledge [1,2,3]

In learning the limit of the sequence is sometimes used the symbol $x_n \to x$ which indicates the intuitive idea that the value of $x_n$ "approximates" the number $x$ if $n \to \infty$ [4]. For example when we pay attention to the sequence of real numbers $\left(\frac{1}{n}\right)$. If $n$ is enlarged then the denominator will become larger so that the result for it will be smaller towards the number 0. Another example, for $\left(\frac{x}{e^x}\right)$ we can intuitively consider how fast $e^x$ enlarged is compared to $x$. Because for a large enough $x$ , $e^x$ rise faster than $x$ then the row of the sequence will be smaller value to 0. Thus there is an intuitive reason for a sequence is said to be convergent. So, in learning the convergence of sequence need to have thought intuitively that will facilitate in determining convergence of sequence before calculating its limit value. Nardi et al [5] discussed how pedagogical practice can guide the students' perceptions of resilience, such as if the term of sequences is smaller the series is convergent. Thus we obtained our convergent intuition through the verbal statement. Intuition is needed to determine the truth value of a mathematical statement and produce evidence and examples of deniers [6].

The term intuition has many meanings and has caused much controversy. The general public sometimes views intuition as a hunch, guess, even as a supernatural ability. In contrast to the general public's view, intuition is a product of previous experience and reasoning (Westcott, in [6]) through active experience and knowledge construction [7,8]. Intuition can arise from previous experiences in the form of intellectual exercises called secondary intuition, and can also develop in human nature naturally, not obtained from the learning process, called primary intuition [6]. Until now, there has not been a firm agreement on the definition of intuition and how intuition works [9]. The definition of intuition relies heavily on the field of study [10].

In mathematics, intuition is used with two rather different meanings. On the one hand, the individual is said to think intuitively if he has been working for a long time on a problem, suddenly he gets the solution, where he has provided a formal proof. On the other hand, the individual is called a good intuitive mathematician if others ask him questions, he can either quickly guess something, or which of the several approaches to a problem that proves successful [11]. Fischbein [6] examined the formation of individual student concepts and elaborated how the intuitive model influences mathematical learning states that formal concepts and statements are very often associated in one's mind, with certain examples. Such specific examples could be universal representatives of each concept and statement and then acquire the heuristic attributes of the model, but these are often ignored.

Associated with problem solving, Dehaene [12] who conducted cognitive neuroscience research, said mathematicians often bring up his intuition when solving problems quickly and automatically. Intuition plays an important role in finding solutions to problems and in understanding the content of the problem [13] and in guiding to find solutions to scientific problems [14]. When tasks can not be performed analytically, requiring pattern recognition, complexity, and high time pressure, intuitive thinking may be more profitable [15].

Until now, intuition is still an interesting thing to be studied. Intuition studies are commonly found in the fields of philosophy, psychology [16,17], mathematics education [13,18,19,20], economics [21], biology & health [22]. Based on his review of existing intuition models, Welling [14] proposed a model of intuitive knowledge representation associated with cases in psychotherapy. This paper presents an intuitive thinking process in every stage of mathematical problem solving based on the results of research on the students.

This study is part of a larger study that investigates how intuitive thinking processes in determining the sequence convergence. In general, the study obtained two categories of intuitive thinking based on the process of problem-solving intuitively, that is straight thinking intuitively and tortuous thinking intuitively. The student is said to think straight intuitively if he solves the problem according to his path without ever changing the flow of his mind. While the student is said to think tortuous intuitively if he experienced several deadlocks in solving the problem so that he changed the flow of his mind. This paper focuses on tortuous thinking intuitively in determining the sequence convergence.

## 2. Research Method

This qualitative research is conducted to explore the intuitive thinking process of students, especially students who have experienced several deadlocks in thinking so that he changed the flow of his mind.

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The study was conducted on 30 students of Mathematics Education FKIP Unila who have ever followed the Real Analysis lecture. Of the number of students given the assignment, there are six students who use intuitive thinking. Of the six people were selected 2 people who use the intuitive winding thought process as the subject of research; but only one subject is described here because both have similar characteristics. The research data were obtained from visual audio recording from think aloud and interview to solve the problem of convergence of sequence by students.

The task given is as follows.

How do you think the sequence is convergent or divergent?

$$\left(\frac{\sin 2n}{4n}\right)$$

## 3. Result and discussion

Subjects were given problems about the sequence convergence. Not long after the problem is given, the subject responds to the problem. "In my opinion, because of its format like this, I use trial and error first". Then the subject writes:

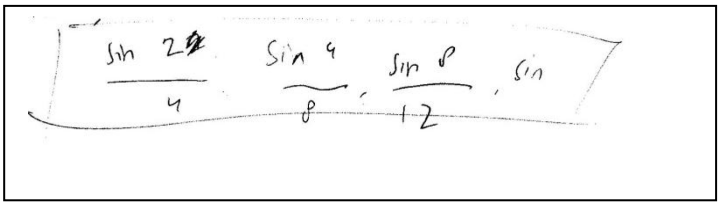

**Figure 1.** Student's counting by trial and error

He stagnated, "if this ........ ."The subject paused for a moment then continued, " ....o, if we find the limit.....," then he wrote

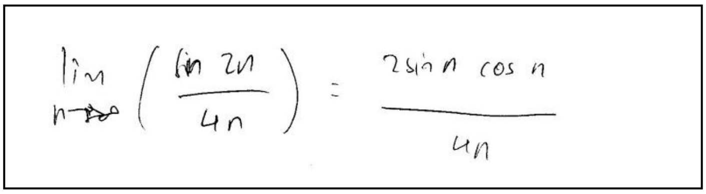

**Figure 2.** Use of Trigonometric equation by subject

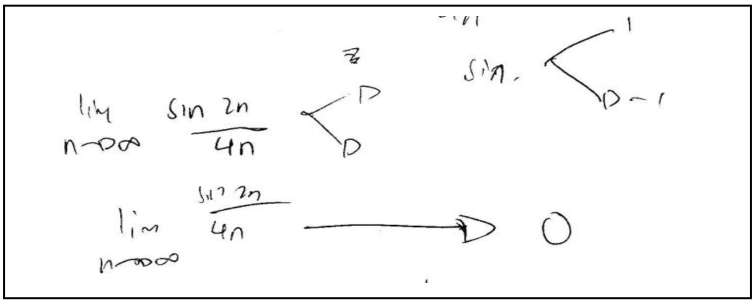

**Figure.3.** Dramatization by the subject

He added, "if so......... owyeah .... the limit of sin2n / 4n ........ umm ...... we know that sine, in trigonometry, the value is only maximum 1 and minimum -1, regardless n its, like that. The problem only here, if we combine the same bottom. My initial assumption is that this will also be a convergent sequence, where it will converge to 0. As it is. I just might not be too pushy by using a substitution like that". While saying that, the subject writes as at Fig 3..

Based on the work of the subject through think aloud, it appears that the Subject experienced three times the process of planning and implementing problem solving that appears intuitively in solving the problem. It is said to appear intuitively because the process came up suddenly, immedately, when the subject encountered a problem. A person can answer immediately, if he identifies the elements in the problem self-evident, and he has the memory of those elements by intrinsic certainty that will be immediately matched to the problem even without being properly planned in order to avoid any interference [23].

When solving problem, firstly, the subject tries by trial and error but stagnating; Secondly, the subject tries to solve the problem again by using the limit but again stagnating; and thirdly, the subject solves the problem by looking at configuration or behavior of the term of sequence. From the interview, the subject reveals the reason as follows, "Well maybe because if not using a calculator, it is hard to determine the value of sin2, sin 4, sin 8. Then it occurred to me ....emm ... what if modified, like ... its trigonometric function is disparted. It turns out well ... emm ....yes ... distress too. Then I turned again with ... assuming that the sine value, or the optimum value of sine and cosine functions is 1 and -1. Well, this will be 1 per 4n where n goes up to infinity and -1 per 4n where n goes down to infinity. If such a certain value would lead to a point, to a number or perhaps to 0; if it seems to me that way." It appears that at the beginning, the subject relied solely on the first thing he remembered (primacy effect) in planning the problem solving [6].

Subjects plan problems quickly and tend to use trial and error. When the subject solves the problem by trial and error and then uses a limit, he does so in the familiar pattern. In general, intuition is based on pattern recognition [24]. Welling [14] intuition is important in the process of knowledge construction, guiding to find solutions to a problem, and giving a promising direction when facing a deadlock in finding a solution. Based on pattern recognition, intuition comes in two forms: (1) recognition of known patterns, or (2) diversion from the general pattern [1]. When Subject gets stuck several times, he switches by using a pattern not commonly used in strict problem solving. This procedure is known as a heuristic procedure which is a strict method of achieving a problem solution. Heuristic procedures are often available when there are no known algorithmic procedures, and even even when algorithms are available, heuristic procedures are often much faster. [11]

In the third cycle, subject solves the problem by looking at the form or behavior of the term of sequence. From the interview, "I assumed that the denominator, 4n is bigger than the numerator, sin 2n. We know that the sinus is a periodic function in which the maximum value is 1 and -1. While 4n, it kept getting bigger, 4, 8, 12, the greater hold. While sin 2n remains maximum 1 or -1 ". From the answer revealed that the subject did a dramatization of combining some facts into a logical unity [6]. When it comes to n = 3 students stop counting and make guesses to draw conclusions, we say students meet one of the intuitive thinking characteristics of 'premature closure' ie immediately ending the search for new information and examination of the argument [6].

Such an answer is not the answer that should be done in the rigorous lecture of real analysis, perhaps even the answer is not perfect in shape but the student has been able to create meaningful mathematical structures [25]. Such an answer may arise possibly because the teacher provides the opportunity for students to respond flexibly and productively to the problem with the student's own way [26]. In addition, the emphasis on the structure or interconnection between knowledge will improve the facilitation of students' intuitive thinking [11]. Besides that, it can be arise because the student is in formal thinking level. According Rahman and Ahmar [29] student in formal thinking are able to think many possibilities which cause the problem representation can be associated with problem context.

In the next step the subject is guessing the solution by saying, "My initial assumption, this is also a convergent sequence, where it converges to 0." The subject shows extrapolative characteristics after paying attention to form or line behavior. Extrapolation is the essence of intuition, its capacity is to make conjectures even without empirical support [27] and based on a limited amount of information [28]. In the final stage, the subject states with complete sentences about the solution.

## 4. Conclusion

Based on result and discuss, the tortuous thinking intuitively in solving problem described as follows steps. Student : 1) understands the problem self-evidently and accepts the existing statement in the issue with intrinsic certainty and with great confidence, 2) plan the problem solving immediately and tend to rush, through trial and error based on primacy effect, 3) carry out the plan in order to get behavior of the sequence's term; however he is stuck. Then the students go back to step 2) plan the problem solving and then continue to step 3). This can last several times. After finding the behavior of the term of sequence, students go to the next step that is 4) make guesses about solution, 5) declare the solution with complete sentence.

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